

Comparing approaches to pro-completions

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While Grothendieck has introduced ind- and pro-objects via (co)presheaves, and category theorist view them as objects in a categorical (co)completion characterized by its universal property, the shape theory community has predominantly been using a language in which specific choices of inverse systems and representatives of their morphisms are carried throughout the development of the theory. While this has an advantage in the treatment of many concrete questions, for some theoretical questions it may be simpler to use the original approach. Knowing the translation between the languages is also useful to import relevant results from categorical community from the classical ones like the structure of the Kock-Zöberlein monad on the cocompletion functor (relevant to the treatment of iterated systems) to the modern ones like the reformulation of strong shape theory in terms of $(\infty, 1)$ -categories due Guenther and then placed into the framework of pro-completions of $(\infty, 1)$ categories and of $(\infty, 1)$ -topoi more recently by Lurie. At the end I will present my rudimentary ideas why known results about $(\infty, 1)$ -topoi should lead to a version of Blakers-Massey theorem for strong shape, paralleling the classical Massey theorem in homotopy theory as well as the version for ordinary shape due Ungar.