Unifying large scale and small scale geometry

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A topology on a set X is the same as a projection (i.e. an idempotent linear operator) $cl : 2^X \to 2^X$ satisfying $A \subset cl(A)$ for all $A \subset X$. That's a good way to summarize Kuratowski's closure operator.

Basic geometry on a set X is a dot product $\cdot : 2^X \times 2^X \to 2^Y$. Its equivalent form is an orthogonality relation on subsets of X. The optimal case is if the orthogonality relation satisfies a variant of parallel-perpendicular decomposition from linear algebra.

We show that this concept unifies small scale (topology, proximity spaces, uniform spaces) and large scale (coarse spaces, large scale spaces). Using orthogonality relations we define large scale compactifications that generalize all well-known compactifications: Higson corona, Gromov boundary, Čech-Stone compactification, Samuel-Smirnov compactification, and Freudenthal compactification.

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